Exam. Code : 211004
Subject Code : 4641

**M.Sc. (Mathematics) 4th Semester**

**DISCRETE MATHEMATICS—II**

**Paper—Math-585**

Time Allowed—Two Hours] [Maximum Marks—100

**Note** :—Attempt any **FOUR** questions. All questions carry equal marks.

1. (a) Let L be the set of all factors of 12 and let '/' be the divisibility relation on L. Show that (L, '/') is a lattice.

   (b) Define dual of a lattice. Show that dual of a lattice is a lattice.

2. (a) Let $(L \leq)$ be a distributive lattice. Show that if $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ for some $a \in L$, then $x = y$.

   (b) Prove that two bounded lattices $L_1$ and $L_2$ are complemented if and only if $L_1 \times L_2$ is complemented.

3. (a) Let B = {1, 2, 3, 6} be the set of positive factors of 6. Let the binary operations '+' and '*' on B are defined as follows :

$a + b = \text{l.c.m}\,(a, b)$ and $a * b = \text{g.c.d.}\,(a, b)$, $\forall\, a, b \in B$.

Let the unary operation ',' on B is defined as

$a' = \frac{6}{a} \;\forall\; a \in B$.

Show that {B, +, *, '} is a Boolean algebra.

(b) Write all the postulates of a Boolean algebra. Using these Boolean postulates, simplify the following expressions :

$f(a, b, c, d) = a + ab + abc + abcd + a' + a'b + a'bc + a'bcd.$

4. (a) Consider the following Boolean expressions :

(i) $f_1(x, y, z) = z(x' + y) + y'$,

(ii) $f_2(x, y, z) = x(xy' + x'y + y'z)$.

Reduce these expressions to sum-of-products form and hence to complete sum of products form.

(b) Consider the following Boolean expressions :

(i) $f_1(x, y, z) = (xy' + xz)' + x'$

(ii) $f_2(x, y, z) = (x'y)'\,(x + z)$.

Reduce these expressions to product to sums form.

5. (a) A graph G has 21 edges, 3 vertices of degree 4 each and other vertices are of degree 3 each. Find the number of vertices in G.

(b) Show that there does not exists a graph with 5 vertices having degrees : 1, 3, 4, 2, 3 respectively.

(c) Determine whether it is possible to construct a graph with 12 edges such that 2 of the vertices have degree 3 each and the remaining vertices have degree 4 each.

(d) Show that the maximum number of edges in a graph with n vertices and no multiple edges are $\frac{n(n-1)}{2}$.

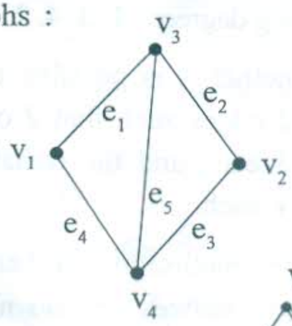
6. (a) Draw the undirected graphs corresponding to each of the following adjacency matrices :

(i) $A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

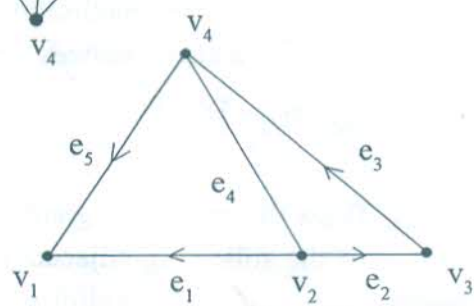
(ii) $A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$

(b) Obtain the incidence matrix for each of the following graphs :

(i)




(ii)




7. (a) Show that the chromatic number of a graph $c_n$, where $c_n$ is the cycle graph with n vertices, is either 2 or 3.

(b) State and prove Euler's theorem on graphs.

8. (a) Prove that a simple graph is connected iff it has a spanning tree.

(b) Determine the minimum spanning three of the weighted graph as shown below :